

OPTIMAL RECOVERY IN HARDY-SOBOLEV SPACES AND AN ANALOGUE OF SPLINES FOR ANALYTIC FUNCTIONS

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Denote by H_∞^r the class of analytic in the unit disk D functions f for which $|f^{(r)}(z)| \leq 1$, $z \in D$. We consider the problem of optimal recovery of $f(\tau)$, $\tau \in (-1, 1)$, using the information $If = (f(t_1), \dots, f(t_{n+r}))$, $t_j \in (-1, 1)$. We calculate the value

$$e(\tau, H_\infty^r, I) = \inf_{S: \mathbb{C}^{n+r} \rightarrow \mathbb{C}} \sup_{f \in H_\infty^r} |f(\tau) - S(If)|$$

and find an optimal algorithm S_0 for which the infimum is attained.

For every system of points $-1 < t_1 < \dots < t_{n+r} < 1$ there exist such $-1 < x_1 < \dots < x_n < 1$ that the function

$$g \in X_{n+r} = \text{span}\{1, z, \dots, z^{r-1}, g_1(z), \dots, g_n(z)\},$$

where

$$g_j(z) = \int_0^z \frac{(z-t)^{r-1}(1-t^2)\omega_j(t)}{(r-1)!(1-x_j t)^2} dt, \quad \omega_j(t) = \prod_{\substack{k=1 \\ k \neq j}}^n \frac{t-x_k}{1-x_k t},$$

which interpolates f at the points t_1, \dots, t_{n+r} gives an optimal method of recovery $f(\tau) \approx g(\tau)$ for all $\tau \in (-1, 1)$. Thus the space X_{n+r} is an analogue of polynomial splines which appear in the similar problem for the Sobolev classes.